Section 35. The Tietze Extension Theorem

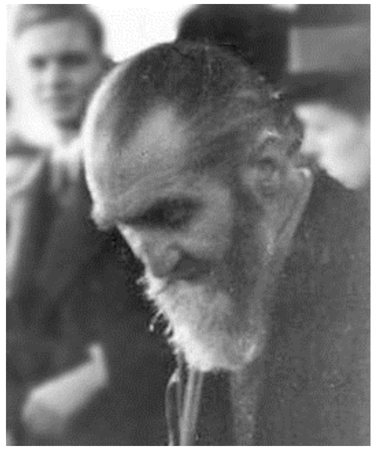
Note. The Tietze Extension Theorem deals with the extension of a continuous function from a closed subspace of a regular space to the whole space. It is a consequence of the Urysohn Lemma (Theorem 33.1), and if we assume the Tietze Extension Theorem then we can prove the Urysohn Lemma (see Exercise 35.1).

Theorem 35.1. The Tietze Extension Theorem.
Let X be a normal space. Let A be a closed subspace of X.

(a) Any continuous function of A into the closed interval $[a, b] \subset \mathbb{R}$ may be extended to a continuous function on all of X into $[a, b]$.

(b) Any continuous function of A into $\mathbb{R}$ may be extended to a continuous function on all of X into $\mathbb{R}$.

Note. The result of this section is sometimes called the Tietze-Urysohn Theorem or the Urysohn-Brouwer Lemma. This is because it was proved by L. E. J. Brouwer (1881–1966) and Henri Lebesgue for $\mathbb{R}^n$, by Heinrich Tietze (1880–1964) for metric spaces, and Pavel Urysohn (1898–1924) for normal topological spaces. This information is from `https://www.encyclopediaofmath.org/index.php/Urysohn-Brouwer_lemma`, a website maintained by Springer-Verlag and The European Mathematical Society.



Heinrich Tietze; August 31, 1880–February 17, 1964

Image from the MacTutor History of Mathematics archive website

Heinrich Tietze was awarded his doctorate in 1904 from Technische Hochschule in Vienna, Austria. Soon after, he developed an interest in topology and became a professor of math studying topological invariants. He served in the Austrian army during World War I. He was the chair of math at the University of Erlangen from 1919 to 1925, then became chair at the University of Munich. He lived in Munich until has death. He contributed to the foundations of general topology and did the bulk of his work after 1925. He created the "Tietze transformations" and used it to show that fundamental groups (which we introduce in Chapter 9) are topological invariants. He also worked on knot theory, Jordan curves, and continuous mappings of areas. Outside of topology, he studied continued fractions, the distribution of prime numbers, and differential geometry. This biographical information is from `http://www-history.mcs.st-and.ac.uk/Biographies/Tietze.html`.

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