

Section 42. The Smirnov Metrization Theorem

Note. Recall that the Nagata-Smirnov Metrization Theorem (theorem 40.3) states that a space is metrizable if and only if it is regular and has a basis that is countably locally finite. In this section we give another necessary and sufficient condition for metrizability, this time involving paracompactness. First, we need a definition.

Definition. A space X is *locally metrizable* if every point $x \in X$ has a neighborhood U that is metrizable in the subspace topology.

Note. Of course, a metrizable space is locally metrizable.

Theorem 42.1. The Smirnov Metrization Theorem.

A topological space X is metrizable if and only if it is a paracompact Hausdorff space that is locally metrizable.

Revised: 10/29/2016