Chapter 1. Groups

Note. Ideally a complete study of groups would culminate in a classification of all groups, both finite and infinite. Naturally, this would lead to a classification of isomorphic groups. An analogy of the idealized result is available for vector spaces as follows:

1. A vector space of dimension n with scalar field $\mathbb{F}$ is isomorphic to $\mathbb{F}^n$ (this is sometimes called *The Fundamental Theorem of Finite Dimensional Vector Spaces*).

2. An infinite dimensional vector space over field $\mathbb{R}$ (with a few more technical details) is isomorphic to ℓ_2 (this is *The Fundamental Theorem of Infinite Dimensional Vector Spaces*—see *Real Analysis with an Introduction to Wavelets*, Don Hong, Jianzhong Wang, and Robert Gardner, Academic Press/Elsevier Press, 2005).

Unfortunately it does not seem likely that a parallel result will follow for groups. Hence, classification of various *kinds* of groups is desired. For example, there is a complete classification of finitely generated abelian groups (see Theorem II.2.1). As a result of many years of study, there is also a classification of finite simple groups. For details on the classification, see my online notes for Introduction to Modern Algebra (MATH 4127/5127) on Supplement. Finite Simple Groups. The proof involved hundreds of papers and thousands of pages of journal articles. The proof was completed around 1980, and so is more recent than the Hungerford textbook, which is copyright 1974.

Note. In this chapter, we quickly review some of the results of group theory from your undergraduate group theory class.

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