

# Chapter IV. Modules

**Note.** Modules over a ring are a generalization of abelian groups. Certain types of modules over a division ring are vector spaces. As a consequence, as Hungerford says, “They are basic in the further study of algebra” (page 168).

**Note.** Galois theory and extension fields are strongly based on the use of vector spaces. In order to cover Chapter V, Fields and Galois Theory, we need a couple of results from this chapter (namely, Theorem IV.2.5 and Theorem IV.2.16). Since we are likely to prioritize Chapter V over this chapter, we may attempt to skip this chapter.

**Note.** Hungerford claims that this chapter is also required for Chapters VII, VIII, and IX. In particular, if we want a deeper study of rings (which is given in Chapter VIII, Commutative Rings & Modules, and Chapter IX, Structure of Rings) then the first step is to cover this chapter on modules.

**Note.** You may have noticed that neither John Fraleigh (in his *A First Course in Abstract Algebra*, 7th edition) nor Joseph Gallian (in his *Contemporary Abstract Algebra*, 8th edition) mention modules in their undergraduate texts (though both have chapters on vector spaces).

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