Chapter I. Groups
Study Guide

The following is a brief list of topics covered in Chapter I of Hungerford's *Algebra*. This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the proofs given in class and the homework problems.

Section I.1. Semigroups, Monoids, and Groups.
Binary operation, associative, identity, inverse, semigroup, monoid, group, abelian/commutative, order of a group, finite group, binary algebraic structure, left and right cancellation, solutions to $ax = b$ (Proposition I.1.4), D_4^*, S_n, permutation notation, direct product and direct sum of two groups, congruence relation, $\mathbb{Z}_m$, meaningful product, standard product, Generalized Associative Law (Theorem I.1.6), Generalized Commutative Law (Corollary I.1.7).

Section I.2. Homomorphisms and Subgroups.
homomorphism, monomorphism, epimorphism, isomorphism, endomorphism, automorphism, Kernel, image, inverse image, one to one homomorphism and kernel (Theorem I.2.3(i)), closed under a binary operation, subgroup, trivial subgroup, proper subgroup, establishing a subgroup (Theorem I.2.5), subgroup generated by a set, generators, finitely generated, cyclic groups, what the elements of $G = \langle X \rangle$ look like (Theorem I.2.8), the join of two groups ($H \vee K$).

Section I.3. Cyclic Groups.
Order of an element of a group, properties of the order of an element (Theorems I.3.4, I.3.5, and I.3.6).

Section I.4. Cosets and Counting.
right an left congruence modulo a subgroup, right and left cosets (Theorem I.4.2), properties of cosets (Corollary I.4.3), the index of subgroup H in group G $[G : H]$, Theorem I.4.5, Lagrange's Theorem (Corollary I.4.6), counting arguments (Theorem I.4.7, Proposition I.4.8, Proposition I.4.9).

Section I.5. Normality, Quotient Groups, and Homomorphisms.
Normal subgroup (Theorem I.5.1), subgroups of index 2 (Exercise I.5.2), normal subgroups and joins (Theorem I.5.3), coset multiplication based on normal subgroups is well-defined (Theorem I.5.4),

quotient group (factor group), the canonical epimorphism (Theorem I.5.5), the three isomorphism theorems (Corollary I.5.7, Corollary I.5.9, Corollary I.5.10).

Section I.6. Symmetric, Alternating, and Dihedral Groups.
Cyclic permutations, transposition, disjoint permutations, elements of S_n are products of disjoint cycles (Theorem I.6.3), even and odd permutations, the sign of a permutation, an element of S_n cannot be both even and odd (Theorem I.6.7), the alternating group A_n, simple group, A_n is simple if and only if $n \neq 4$, the dihedral group D_n (Theorem I.6.13).

Section I.7. Categories: Products, Coproducts, and Free Objects.
Category, morphism, composite morphism, equivalent morphisms and categories, product of categories, products of the same family are equivalent (Theorem I.7.3), coproduct, concrete category, free object on a set, universal and couniversal objects, equivalence of universal(couniversal) objects (Theorem I.7.10).

Section I.8. Direct Products and Direct Sums.
Direct product of groups, canonical projection, Theorem I.8.2, weak direct product (sum) of groups, canonical injection, Theorem I.8.5, normal subgroups and weak direct products (Theorem I.8.6), internal weak direct product of normal subgroups, products of normal subgroups (Corollary I.8.11).

Section I.9. Free Groups, Free Products, Generators & Relations.
Word, reduced word, equal words, free group on a set, Theorem I.9.2, every group is the homomorphic image of a free group (Corollary I.9.3), group presentations (generators and relations), von Dyck's Theorem (Theorem I.9.5), word and reduced word on groups.

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