Chapter III. Rings

Study Guide

The following is a brief list of topics covered in Chapter III of Hungerford's *Algebra*. This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the proofs given in class and the homework problems.

Section III.1. Rings and Homomorphisms.
Ring, commutative ring, ring with identity, elementary properties of a ring (Theorem III.1.2), left/right zero divisor, zero divisor, no zero divisors if and only if left or right cancellation hold (Lemma III.1.A), left/right invertible, left/right inverse, unit, integral domain, division ring, field, group ring of a multiplicative group over a ring, the real quaternions as a noncommutative division ring, Binomial Theorem (Theorem III.1.6), ring homomorphism, kernel of a homomorphism, characteristic of a ring, properties of the characteristic of a ring (Theorem III.1.9), embedding a ring in a ring with identity (Theorem III.1.10).

Supplement. Quaternions—An Algebraic View.
Ring, commutative ring, ring with identity, left/right zero divisor, zero divisor, the real quaternions $\mathbb{H}$, integral domain, division ring, field, the quaternions form a noncommutative division ring ("Theorem"), Cayley digraph of the "quaternion group"the quaternions as 2×2 complex matrices, complex numbers as ordered pairs of real numbers, William Rowan Hamilton and the quest for a three dimensional number system, The Factor Theorem for commutative rings, distinct roots of a polynomial over an integral domain (Hungerford's Theorem III.6.7), infinite number of square roots of -1 in the quaternions, $\mathbb{S} = \{q = x_1 i + x_2 j + x_3 k \mid x_1^2 + x_2^2 + x_3^2 = 1\}$, $s + y\mathbb{S}$, quaternionic polynomials, the product of quaternionic polynomials, zeros of a quaternionic polynomial involving $\mathbb{S}$ ("Theorem"), left/right root of a polynomial, The Factor Theorem in a Ring with Unity (Proposition 16.2 of Lam), left/right roots of a product of polynomials over a division ring (Proposition 16.3 or Lam), quaternionic conjugate, left/right roots of a polynomial over a division ring occur in conjugacy classes (Theorem 16.4 of Lam), modulus, left/right algebraically closed division ring, The Fundamental Theorem of Algebra for Quaternions (Theorem 16.14 of Lam), the structure of the set of roots for a quaternionic polynomial of degree n (Theorem of Pogorui and Shapiro).

Section III.2. Ideals.
Subring, left/right ideal, ideal, proper ideal, conditions for a subset of a ring to form an ideal (Theorem III.2.2), an intersection of ideals is an ideal (Corollary III.2.3), ideal generated by a set

X, generators, finitely generated ideal, principal ideal, principal ideal ring, principal ideal domain ("PID"), the structure of principal ideals and ideals generated by a set (Theorem III.2.5), sums and products of subsets of a ring, properties of sums and products of subsets of a ring (Theorem III.2.6), quotient rings of a ring and ideal (Theorem III.2.7), kernels of homomorphisms and ideals (Theorem III.2.8), First/Second/Third Isomorphism Theorems for quotient rings (Corollary III.2.10 and Theorem III.2.12), prime ideal, classification of prime ideals (Exercise III.2.14), classification of prime ideals in commutative rings (Theorem III.2.15), prime ideals and integral domain quotients (Theorem III.2.16), maximal left/right ideal, every proper ideal is contained in a maximal ideal (Theorem III.2.18), $R^2 = R$ for commutative ring R implies every maximal ideal is prime (Theorem III.2.19), maximal ideals and quotients that are division rings or fields (Theorem III.2.20), conditions equivalent to a commutative ring being a field (Corollary III.2.21), direct product of rings (Theorem III.2.22), a direct product of rings is a product in the category of rings (Theorem III.2.23), conditions that give a ring as an internal direct product (Theorem III.2.24), $a \equiv b \pmod{A}$ where A is an ideal in a ring R, Chinese Remainder Theorem (Theorem III.2.25), solutions of a collection of congruences (Corollary III.2.26), monomorphism of quotient rings and direct products (Corollary III.2.27).

Section III.3. Factorization in Commutative Rings.

An element in a commutative ring divides another element, associates in a ring, properties of associates and units in a commutative ring with identity (Theorem III.3.2), irreducible element of a commutative ring with identity, prime element of a commutative ring with identity,

Section III.4. Rings of Quotients and Localization.

.

Section III.5. Rings of Polynomials and Formal Power Series.

.

Section III.6. Factorization in Polynomial Rings.

.

Revised: 12/22/2023