Chapter V. Fields and Galois Theory

Note. This chapter contains the most important results of Modern Algebra 2 (MATH 5420): The Fundamental Theorem of Algebra (Theorem V.3.19 in the appendix to Section V.3) and The Unsolvability of the Quintic (Proposition V.9.8 in the appendix to Section V.9).

Note. As stated on page x, Hungerford's treatment of Galois theory is based on the approach of Irving Kaplansky (*Fields and Rings*, 2nd Edition, University of Chicago Press, 1972) who extended the ideas of Emil Artin (stated in Artin's *Galois Theory*, Notre Dame Mathematical Lectures No. 2, 2nd Edition, 1944—this is still in print by Dover Publications [1998] and is about $8.00 [as of 2015]; you may find a copy online at such places as GoogleBooks).

Note. In Galois theory, we consider field F an extension field of field K (that is, K is a subfield of field F). The Galois group of extension F of K is the group of all automorphisms of F that fix K elementwise (Fraleigh called this "leaving K fixed"—see Fraleigh's Section X.48 in the 7th edition). The Fundamental Theorem of Galois Theory (Theorem V.2.5) states that there is a one-to-one correspondence between the intermediate fields of a (finite dimensional) Galois field extension and the subgroups of the Galois group of the extension. For a more elementary presentation of this material, see Part X of Fraleigh's 7th edition (Chapters 48 to 56).

Note. The fundamental theorem allows us to translate problems involving fields, polynomials, and field extensions into group theoretic terms (thus making group theory a central part of modern algebra as well as classical algebra—particularly the algebraic solvability of a polynomial equation as stated in Corollaries V.9.5 and V.9.7).

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