Proposition VI.2.2

Let \( f \) be a continuous function on \( D \), where \( D \) is an open disk in the complex plane. Then, \( f \) is analytic if and only if \( f \) is continuous on \( D \) and \( f \) is analytic at \( 0 = z \). Since \( f \) is continuous at \( 0 = z \) and \( f \) is analytic on \( D \), it is also continuous on \( D \). Consequently, \( f \) is analytic on \( D \).

Proof.

Let \( f \) be a function on \( D \), where \( D \) is an open disk in the complex plane. Then, \( f \) is analytic if and only if \( f \) is continuous on \( D \) and \( f \) is analytic at \( 0 = z \). Since \( f \) is continuous at \( 0 = z \) and \( f \) is analytic on \( D \), it is also continuous on \( D \). Consequently, \( f \) is analytic on \( D \).

Lemma VI.2.1

Let \( D \) be an open disk in the complex plane. Then, \( f \) is analytic on \( D \) if and only if \( f \) is continuous on \( D \) and \( f \) is analytic at \( 0 = z \). Since \( f \) is continuous at \( 0 = z \) and \( f \) is analytic on \( D \), it is also continuous on \( D \). Consequently, \( f \) is analytic on \( D \).

Proof.

Let \( f \) be a function on \( D \), where \( D \) is an open disk in the complex plane. Then, \( f \) is analytic if and only if \( f \) is continuous on \( D \) and \( f \) is analytic at \( 0 = z \). Since \( f \) is continuous at \( 0 = z \) and \( f \) is analytic on \( D \), it is also continuous on \( D \). Consequently, \( f \) is analytic on \( D \).
Lemma V.2.A (continued 2)

Proof (continued).

Lemma V.2.A (continued 1)

Proof (continued).

Proposition V.2.2 (continued)

Proposition V.2.2 (continued).
\[ \frac{ze - 1}{e - z} \mid W = \mid (z)^e \mid \] \[ \mid z \mid W \leq \mid \frac{(ze - 1)/(e - z)}{\mid z \mid} \mid \]

\begin{proof}

Define \( W \) by

\[ \mid (z)^e \mid \]

\[ \frac{ze - e}{e - z} \mid W \leq \mid (z)^e \mid \]

Then for \( z \in D \)

\[ \mid z \mid > 1 \text{ where } 0 = (e)^f (0) \]

and \( D \) is analytic on \( \{ z \mid |z| > 1 \} \).

\end{proof}

**Generalized Schwartz's Lemma**

Theorem V.2.5. Let \( f : D \rightarrow D \) be a one to one analytic map of \( D \) onto \( D \) such that \( f(c) = c \).

Then there is a complex \( c \) where \( \mid c \mid = 1 \) such that \( f(z) = z \).

**Proof**

Apply Proposition 3.2.0. Applying Lemma V.2.4 to both \( f \) and \( f^* \) to both \( f \) and \( f^* \) to both \( f \) and \( f^* \).

Since \( f \) is one to one and onto, then there is such that \( f(z) = z \).

**Theorem V.2.5 (continued)**

Theorem V.2.5. Let \( f : D \rightarrow D \) be a one to one analytic map of \( D \) onto \( D \) such that \( f(c) = c \).

Then there is a complex \( c \) where \( \mid c \mid = 1 \) such that \( f(z) = z \).

**Proof**

Apply Proposition 3.2.0. Applying Lemma V.2.4 to both \( f \) and \( f^* \) to both \( f \) and \( f^* \) to both \( f \) and \( f^* \).

Since \( f \) is one to one and onto, then there is such that \( f(z) = z \).