Complex Analysis 1, MATH 5510, Spring 2022

Homework 1, Sections I.2 and I.3

Due Tuesday, January 25 at 3:45

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses.

I.2.1. (b) Find the real and imaginary parts of $(z-a)/(z+a)$, where $a \in \mathbb{R}$, in terms of $\mathrm{Re}(z)$, $\mathrm{Im}(z)$, and moduli.

I.2.1. (e) Find the real and imaginary parts of $((-1+i\sqrt{3})/2)^3$.

I.2.A. Prove $\overline{z/w} = \overline{z}/\overline{w}$ using real and imaginary parts of z and w.

I.2.6. Let $R(z)$ be a rational function of z. Prove that if all the coefficients of $R(z)$ are real then $\overline{R(z)} = R(\overline{z})$. HINT: By Corollary I.2.A/Exercise I.2.A, for any $z, w \in \mathbb{C}$, we have $\overline{z/w} = \overline{z}/\overline{w}$.

I.3.1. Prove that for all $z, w \in \mathbb{C}$ that $||z|-|w|| \leq |z-w|$. Give necessary and sufficient conditions for equality. HINT: For the equality condition, consider the Corollary I.3.A in the class notes.