Complex Analysis 1, MATH 5510, Spring 2022

Homework 13, Section VI.3

Due Wednesday, May 4

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses.

VI.1.2. Let G be a bounded region and suppose f is continuous on $\overline{G}$ and analytic on G. Show that if there is a constant $c \geq 0$ such that $|f(z)| = c$ for all $z \in \partial(G)$ then either f is a constant function or f has a zero in G. HINT: Consider $f(z)/c$, apply the Maximum Modulus Theorem and the Minimum Principle (Exercises IV.3.6 and VI.1.1).

VI.1.6. Suppose that both f and g are analytic on $\overline{B}(0;R)$ with $|f(z)| = |g(z)|$ for $|z| = R$. Show that if neither f nor g vanishes in $B(0;R)$, then there is a constant λ, $|\lambda| = 1$, such that $f = \lambda g$. HINT: Consider $f(z)/g(z)$ and beware of the set $P = \{z \mid |z| = R \text{ and } f(z) = g(z) = 0\}$. Use Exercise VI.1.2.

VI.1S.2. **(Bonus)** Prove the Joyal, Labelle, Rahman generalization of the Eneström-Kakeya Theorem: If $p(z) = \sum_{k=0}^{n} a_k z^k$ is a polynomial of degree n with real coefficients satisfying $a_0 \leq a_1 \leq \cdots \leq a_n$, then all the zeros of p lie in $|z| \leq (a_n - a_0 + |a_0|)/|a_n|$. HINT: Consider the function $(1-z)p(z) = f(z) - a_n z^{n+1}$ and mimic the proof of the Eneström-Kakeya Theorem.