Complex Analysis 1, MATH 5510, Spring 2022

Homework 2, Sections I.4 and I.6

Due Tuesday, February 1 at 3:45

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses.

- I.4.1. Find the sixth roots of unity.
- **I.4.6.** Prove that $\varphi(t) = \operatorname{cis}(t)$ is a group homomorphism of the additive group $\langle \mathbb{R}, + \rangle$ onto the multiplicative group $\langle T, \cdot \rangle$ where $T = \{z \mid |z| = 1\}$.
- **I.4.7.** If $z \in \mathbb{C}$ and $\operatorname{Re}(z^n) \geq 0$ for every positive integer n, show that z is a non-negative real number. **Hint.** Suppose not. Let $z = re^{i\theta}$ where $-\pi < \theta < \pi$ and consider two cases: (1) $0 < \theta < \pi$ and (2) $-\pi < \theta < 0$. Get a contradiction with a clean analysis argument.
- **I.6.3.** Which subsets of S correspond to the real and imaginary axes of \mathbb{C} ?