Complex Analysis 1, MATH 5510, Spring 2022

Homework 4 REVISED, Sections II.2 and II.3

Due Saturday, February 19

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses.

II.2.5. (a) Prove that if $F \subset X$ is connected then for every pair of points $a, b \in F$ and for each $\varepsilon > 0$ there are points $z_0, z_1, \ldots, z_n \in F$ with $z_0 = a$, $z_n = b$, and $d(z_{k-1}, z_k) < \varepsilon$ for $1 \leq k \leq n$. Notice the hypothesis of "closed" is not given (or needed) here (Conway includes this as part of the question). HINT: Consider $F' = \{b \in X \mid \text{ there are points } z_0, z_1, \ldots, z_n \in F, \text{ for some } n \in \mathbb{N}, \text{ with } z_0 = a, z_n = b, \text{ and } d(z_{k-1}, z_k) < \varepsilon\}$. Show that $F' \cap F$ is both open and closed in (F, d).

II.2.5. (b) If F is a set which satisfies the above property then F is not necessarily connected, even if F is closed. Give an example to illustrate this. Explain your answer. HINT: Think asymptotes.

II.3.3. Prove that $\text{diam}(A) = \text{diam}(A^-)$. HINT: The $\text{diam}(A) \leq \text{diam}(A^-)$ part is easy. For $\text{diam}(A^-) \leq \text{diam}(A)$, use the fact that for given $\varepsilon > 0$ there are $x', y' \in A^-$ such that $d(x', y') > \text{diam}(A^-) - \varepsilon/2$ (a property of suprema). Also, x' and y' are either in A or limit points of A by Proposition II.3.4(b)