Complex Analysis 1, MATH 5510, Spring 2022

Homework 5, Sections II.2 and II.3, Solutions

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses.

II.2.5. (a) Prove that if $F \subset X$ is connected then for every pair of points $a, b \in F$ and for each $\varepsilon > 0$ there are points $z_0, z_1, \ldots, z_n \in F$ with $z_0 = a$, $z_n = b$, and $d(z_{k-1}, z_k) < \varepsilon$ for $1 \leq k \leq n$. Notice the hypothesis of "closed" is not given (or needed) here (Conway includes this as part of the question). HINT: Consider $F' = \{b \in X \mid \text{ there are points } z_0, z_1, \ldots, z_n \in F, \text{ for some } n \in \mathbb{N}, \text{ with } z_0 = a, z_n = b, \text{ and } d(z_{k-1}, z_k) < \varepsilon\}$. Show that $F' \cap F$ is both open and closed in (F, d).

II.3.2. Let (X, d) be a complete metric space and let $Y \subset X$. Prove that if Y is closed in X then (Y, d) is a complete metric space. HINT: Use Proposition II.3.4(a).

II.3.8. Suppose $\{x_n\}$ is a Cauchy sequence and $\{x_{n_k}\}$ is a subsequence that is convergent. Prove that $\{x_n\}$ is convergent. HINT: Use the Triangle Inequality and a $\varepsilon/2$ argument.