# Complex Analysis 1, MATH 5510, Spring 2022

## Homework 7, Section III.1

Due Saturday, March 12

**Write in complete sentences!!!** *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses.

**III.1.3.** Prove that $\limsup(a_n + b_n) \leq \limsup a_n + \limsup b_n$ and $\liminf(a_n + b_n) \geq \liminf a_n + \liminf b_n$ for bounded sequences of real numbers $\{a_n\}$ and $\{b_n\}$. HINT: Recall that $L = \limsup a_n$ means that for all $\varepsilon > 0$, infinitely many $a_n$ satisfy $a_n \in (L - \varepsilon, L + \varepsilon)$ and only finitely many $a_n$ are greater than $L + \varepsilon$.

**III.1.6a.** Find the radius of convergence of the power series $\sum_{n=0}^{\infty} a^n z^n$ where $a \in \mathbb{C}$, $a \neq 0$. HINT: Use Proposition III.1.4.

**III.1.6b.** Find the radius of convergence of the power series $\sum_{n=0}^{\infty} a^{n^2} z^n$ where $a \in \mathbb{C}$. HINT: Use Proposition III.1.4.

**III.1.6d.** Find the radius of convergence of the power series $\sum_{n=0}^{\infty} z^{n!}$. HINT: Use Theorem III.1.3 and ignore the coefficients which are 0.