Complex Analysis 1, MATH 5510, Spring 2022

Homework 8, Sections III.1 and III.2

Due Saturday, March 26

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses.

III.1.7(a). Show that the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n} z^{n(n+1)}$ is 1. HINT: The nth coefficient of this series is not $(-1)^n/n$. Give details.

III.1.7(b). Discuss the convergence of the series in part (a) for $z = 1$, -1, and i. HINT: Consider the value of $\frac{(-1)^n}{n} i^{n(n+1)}$ for n (mod 4). Consider the partial sums of the series s_n for n even and n odd. Show that the sequences of partial sums $\{s_2, s_4, s_6, \ldots\}$ and $\{s_1, s_3, s_5, \ldots\}$ both converge and have the same limit.

III.2.1. Prove that $f(z) = |z|^2 = x^2 + y^2$ does not have a derivative at $z \neq 0$. HINT: Use the definition of derivative and let h approach 0 along the real axis and the imaginary axis. Compare the results.