Complex Analysis 1, MATH 5510, Fall 2023

Homework 1, Sections I.2 and I.3 (revised)

Due Saturday, September 9 at 11:59 pm

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses. Do not discuss homework problems with others. If you have any questions, then contact me (`gardnerr@etsu.edu`).

I.2.1. (a) Find the real and imaginary parts of $1/z$ in terms of $\text{Re}(z)$, $\text{Im}(z)$, and $|z|$.

I.2.1. (d) Find the real and imaginary parts of $(3+5i)/(7i+1)$.

I.2.2(d) (d) Find the absolute value and conjugate of $(3-i)/(\sqrt{2}+3i)$. Express the conjugate in the form (real part) + i(imaginary part).

I.2.4. (a) Prove that for all $z, w \in \mathbb{C}$ we have $|z+w|^2 = |z|^2 + 2\text{Re}(z\overline{w}) + |w|^2$.

(b) Prove that for all $z, w \in \mathbb{C}$ we have $|z-w|^2 = |z|^2 - 2\text{Re}(z\overline{w}) + |w|^2$.

(c) Prove that for all $z, w \in \mathbb{C}$ we have $|z+w|^2 + |z-w|^2 = 2(|z|^2 + |w|^2)$. NOTE: This result is called the "Parallelogram law" since we can geometrically treat the points 0, z, w, and $z+w$ as the corners of a parallelogram in the complex plane with sides of lengths $|z|$ and $|w|$ and diagonals of lengths $|z+w|$ and $|z-w|$. A result from Euclidean geometry is that the sum of the squares of the lengths of the sides of a parallelogram (here, $|z|^2+|w|^2+|z|^2+|w|^2$) equals the sum of the squares of the lengths of the diagonals of the parallelogram (here, $|z+w|^2+|z-w|^2$). The Parallelogram Law can be easily proved with the Law of Cosines.

I.3.2. Prove that for nonzero $z_1, z_2, \ldots, z_n$ we have $|z_1 + z_2 + \cdots + z_n| = |z_1| + |z_2| + \cdots + |z_n|$ if and only if $z_k/z_\ell > 0$ for any integers k and ℓ where $1 \le k, \ell \le n$.