# Complex Analysis 1, MATH 5510, Fall 2023

## Homework 10, Section IV.5. Cauchy's Theorem and Integral Formula

Due Saturday, December 9 at 11:59 pm

**Write in complete sentences!!!** *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses. Do not discuss homework problems with others. If you have any questions, then contact me (`gardnerr@etsu.edu`).

**IV.5.3.** Let $B_\pm = \overline{B}(\pm 1; 1/2)$, $G = B(0;3) \setminus (B_+ \cup B_-)$. Let $\gamma_1, \gamma_2, \gamma_3$ be curves whose traces are $|z-1| = 1$, $|z+1| = 1$, and $|z| = 2$, respectively. Give $\gamma_1$, $\gamma_2$, and $\gamma_3$ orientations such that $n(\gamma_1; w) + n(\gamma_2; w) + n(\gamma_3; w) = 0$ for all $w \in \mathbb{C} \setminus G$. Explain.

**IV.5.4.** Prove that Cauchy's Integral Formula (Theorem IV.5.4) follows from Cauchy's Theorem (Theorem IV.5.7). NOTE: The proof of Cauchy's Theorem follows easily from Cauchy's Integral Formula (see page 85), so this claim shows that the Integral Formula and Theorem are equivalent. HINT: Define $g(z) = (f(z) - f(a))/(z - a)$ for $z = a$ and $g(a) = f'(a)$ for $a \in G$ and $|z - a| < R$ for "appropriate" $R > 0$. PROVE $g$ is analytic on all of $G$. Apply Cauchy's Theorem to $g$.

**IV.5.7.** Let $\gamma(t) = 1 + e^{it}$ for $t \in [0, 2\pi]$. Find $\displaystyle\int_\gamma \left(\frac{z}{z-1}\right)^n dz$ for all positive integers $n$.