Complex Analysis 1, MATH 5510, Fall 2023

Homework 2, Sections I.3S, I.4, and I.6

Due Saturday, September 16 at 11:59 pm

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses. Do not discuss homework problems with others. If you have any questions, then contact me (`gardnerr@etsu.edu`).

I.3S.1 Prove Corollary 1: If $p(z) = \sum_{k=0}^{n} a_k z^k$ is a polynomial of degree n, then all the zeros of p lie in

$$|z| \geq \min_{1 \leq k \leq n} \frac{|a_0|}{|a_0| + |a_k|}.$$

I.4.4. Use the binomial equation $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$ where $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ and compare the real and imaginary parts of each side of de Moivre's formula to obtain formulae for $\cos(n\theta)$ and $\sin(n\theta)$ in terms of $\sin\theta$ and $\cos\theta$.

I.6.1. Give the details in the derivation of equations (6.7) and (6.8):

$$(d(z,z'))^2 = d(z,z') = \frac{2|z-z'|}{((1+|z|^2)(1+|z'|^2))^{1/2}} \text{ for } z, z' \in \mathbb{C}, \text{ and } d(z,\infty) = \frac{2}{(1+|z|^2)^{1/2}}.$$

HINT: Use equation (6.6) which states that $(d(z,z'))^2 = 2 - 2(x_1 x_2' + x_2 x_2' + x_3 x_3\prime)$.