Complex Analysis 1, MATH 5510, Fall 2023

Homework 3, Section III.1. Power Series

Due Saturday, September 30 at 11:59 pm

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses. Do not discuss homework problems with others. If you have any questions, then contact me (`gardnerr@etsu.edu`).

III.1.2(a). Give the details of the proof of Proposition II.1.6 on the sum of series:

Proposition III.1.6. Let $\sum a_n(z-a)^n$ and $\sum b_n(z-a)^n$ be power series with radius of convergence $\geq r > 0$. The power series $\sum(a_n+b_n)(z-a)^n$ has radius of convergence $\geq r$, and $\sum(a_n+b_n)(z-a)^n = \sum a_n(z-a)^n + \sum b_n(z-a)^n$ for $|a-z| < r$.

III.1.4. Use our definition of $\underline{\lim}\, a_n = \liminf a_n$ and $\overline{\lim}\, a_n = \limsup a_n$ to prove that for any sequence $\{a_n\}$ in $\mathbb{R}$ we have $\liminf a_n \leq \limsup a_n$.

III.1.6(d). Find the radius of convergence of the power series $\displaystyle\sum_{n=0}^{\infty} z^{n!}$. HINT: Use Theorem III.1.3 and ignore the coefficients which are 0.