Complex Analysis 1, MATH 5510, Fall 2023

Homework 5, Section III.2. Analytic Functions

Due Saturday, October 14 at 11:59 pm

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses. Do not discuss homework problems with others. If you have any questions, then contact me (`gardnerr@etsu.edu`).

III.2.9. Suppose that $z_n, z \in G = \mathbb{C} \setminus \{z \mid z \leq 0\}$, $z_n = r_n e^{i\theta_n}$, and $z = re^{i\theta}$ where $\theta, \theta_n \in (-\pi, \pi)$. Prove that if $z_n \to z$ then $\theta_n \to \theta$ and $r_n \to r$. HINT: You need to argue geometrically. Let $\varepsilon > 0$. In the complex plane, $|r - r_n| < \varepsilon$ implies that z_n lies in the annulus $r - \varepsilon < |z| < r + \varepsilon$. The condition $|\theta - \theta_n| < \varepsilon$ means that z_n lies in the sector with sides $\theta - \varepsilon$ and $\theta + \varepsilon$.

III.2.11. Suppose $f : G \to \mathbb{C}$ is a branch of the logarithm and that $n \in \mathbb{Z}$. Prove that $z^n = \exp(nf(z))$ for all $z \in G$.

III.2.14. Suppose $f : G \to \mathbb{C}$ is analytic and that G is open and connected. Prove that if $f(z)$ is real for all $z \in G$, then f is constant. HINT: Use the Cauchy-Riemann equations.