Complex Analysis 1, MATH 5510, Fall 2023

Homework 6, Section IV.1. Riemann-Stieltjes Integrals

Due Saturday, October 28 at 11:59 pm

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses. Do not discuss homework problems with others. If you have any questions, then contact me (`gardnerr@etsu.edu`).

IV.1.2. Prove Proposition IV.1.2(b):

Proposition IV.1.2. Let $\gamma : [a,b] \to \mathbb{C}$ be of bounded variation. Then:

(b) if $\sigma : [a,b] \to \mathbb{C}$ is also of bounded variation and $\alpha, \beta \in \mathbb{C}$ then $\alpha\gamma + \beta\sigma$ is of bounded variation and $V(\alpha\gamma + \beta\sigma) \le |\alpha|V(\gamma) + |\beta|V(\sigma)$.

IV.1.3. Prove Proposition IV.1.7(a):

Proposition IV.1.7. Let f and g be continuous functions on $[a,b]$ and let γ and σ be functions of bounded variation on $[a,b]$. Then for any complex scalars α and β:

(a) $\displaystyle\int_a^b (\alpha f + \beta g)\, d\gamma = \alpha \int_a^b f\, d\gamma + \beta \int_a^b g\, d\gamma.$

HINT: Use Theorem IV.1.4.

IV.1.8. Let γ and σ be the two polygons $[1, i]$ and $[1, 1+i, i]$. Express γ and σ as paths and calculate $\int_\gamma f$ and $\int_\sigma f$ where $f(z) = |z|^2$. Quote appropriate definitions, theorems, and propositions to justify your computations.