Complex Analysis 1, MATH 5510, Fall 2023

Homework 8, Section IV.3. Zeros of Analytic Functions

Due Saturday, November 18 at 11:59 pm

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses. Do not discuss homework problems with others. If you have any questions, then contact me (`gardnerr@etsu.edu`).

IV.3.1. Let f be an entire function and suppose there is a constant M, an $R > 0$, and an integer $n \geq 1$ such that $|f(z)| \leq M|z|^n$ for $|z| > R$. Prove that f is a polynomial of degree $\leq n$. HINT: Consider $|f(z)|$ on $|z| = r$ for $r > R$. Use the Maximum Modulus Theorem—Second Version to get a bound for $|f(z)|$ on $B(0; r)$. Use Cauchy's Estimate to show $f^{(k)}(0) = 0$ for $k > n$ (similar to the proof of Liouville's Theorem).

IV.3.3. Find all entire functions f such that $f(x) = e^x$ for all $x \in \mathbb{R}$.

IV.3.10. Prove that if f and g are analytic functions on a region G such that $\overline{f}g$ is analytic then either f is constant on G or $g(z) = 0$ for all $z \in G$.