Complex Analysis 1, MATH 5510, Fall 2023

Homework 9, Section IV.4. The Index of a Closed Curve

Due Saturday, December 2 at 11:59 pm

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook or hypotheses. Do not discuss homework problems with others. If you have any questions, then contact me (`gardnerr@etsu.edu`).

IV.4.2. Give an example of a closed rectifiable curve γ in $\mathbb{C}$ such that for any $k \in \mathbb{Z}$ there is a point $a \notin \{\gamma\}$ with $n(\gamma; a) = k$. HINT: Use a p-series to create a lot of nested circles which are tangent at a single point and have circumferences which sum to a finite number.

IV.4.3. Let $p(z)$ be a polynomial of degree n and let $R > 0$ be sufficiently large so that p never vanishes in $\{z \mid |z| > R\}$. If $\gamma(t) = Re^{it}$, $t \in [0, 2\pi]$, prove that $\displaystyle\int_\gamma \frac{p'(z)}{p(z)}\,dz = 2\pi i n$. HINT: Use the product rule to calculate $p'(z)/p(z)$ in terms of the zeros of p. Use the definition of winding number.

IV.4.4. Fix $w = re^{i\theta} \neq 0$ and let γ be a rectifiable path in $\mathbb{C} \setminus \{0\}$ from 1 to w. Prove that there is an integer k such that $\displaystyle\int_\gamma \frac{1}{z}\,dz = \log(r) + i\theta + 2\pi i k$.