Complex Analysis 1, MATH 5510, Fall 2017

Homework 1, Sections I.2 and I.3

Due: Friday, September 1 at 1:40

Write in complete sentences!!! Explain what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results and equations from the textbook or from the hypotheses.

I.2.1(b) Find the real and imaginary parts of $(z-a)/(z+a)$, where $a \in \mathbb{R}$, in terms of $\mathrm{Re}(z)$, $\mathrm{Im}(z)$, and moduli.

I.2.1(e) Find the real and imaginary parts of $((-1+i\sqrt{3})/2)^3$.

I.2.4. (a) Prove that for all $z, w \in \mathbb{C}$ we have $|z+w|^2 = |z|^2 + 2\mathrm{Re}(z\overline{w}) + |w|^2$.

(b) Prove that for all $z, w \in \mathbb{C}$ we have $|z-w|^2 = |z|^2 - 2\mathrm{Re}(z\overline{w}) + |w|^2$.

(c) Prove that for all $z, w \in \mathbb{C}$ we have $|z+w|^2 + |z-w|^2 = 2(|z|^2 + |w|^2)$. NOTE: This result is called the "Parallelogram law" since we can geometrically treat the points 0, z, w, and $z+w$ as the corners of a parallelogram in the complex plane with sides of lengths $|z|$ and $|w|$ and diagonals of lengths $|z+w|$ and $|z-w|$. A result from Euclidean geometry is that the sum of the squares of the lengths of the sides of a parallelogram (here, $|z|^2+|w|^2+|z|^2+|w|^2$) equals the sum of the squares of the lengths of the diagonals of the parallelogram (here, $|z+w|^2+|z-w|^2$). The Parallelogram Law can be easily proved with the Law of Cosines.

I.2.A. Prove $\overline{z/w} = \overline{z}/\overline{w}$ using real and imaginary parts of z and w.

I.3.1. Prove that for all $z, w \in \mathbb{C}$ that $||z|-|w|| \le |z-w|$. Give necessary and sufficient conditions for equality. HINT: For the equality condition, consider the Corollary I.3.A in the class notes.