Complex Analysis 1, MATH 5510, Fall 2017

Homework 2, Section I.4

Due: Friday, September 8 at 1:40

I.4.2(c) Calculate the square roots of $\sqrt{3} + 3i$.

I.4.6. Prove that $\varphi(t) = \text{cis}(t)$ is a group homomorphism of the additive group $\langle \mathbb{R}, + \rangle$ onto the multiplicative group $\langle T, \cdot \rangle$ where $T = \{z \mid |z| = 1\}$.

I.4.7. If $z \in \mathbb{C}$ and $\text{Re}(z^n) \geq 0$ for every positive integer n, show that z is a non-negative real number. **Hint.** Suppose not. Let $z = re^{i\theta}$ where $-\pi < \theta < \pi$ and consider two cases: (1) $0 < \theta < \pi$ and (2) $-\pi < \theta < 0$. Get a contradiction with a clean analysis argument.