Complex Analysis 1, MATH 5510, Fall 2017

Homework 4, Section II.2

Due: Thursday, September 28 at 1:40

II.2.4. Let $\{D_j \mid j \in J\}$ be a collection of connected subsets of X, where (X, d) is a metric space, where for each $j, k \in J$ we have $D_j \cap D_k \neq \varnothing$. Prove that $D = \cup\{D_j \mid j \in J\}$ is connected. HINT: Mimic the proof of Lemma II.2.6.

II.2.5. (a) Prove that if $F \subset X$ is connected then for every pair of points $a, b \in F$ and for each $\varepsilon > 0$ there are points $z_0, z_1, \ldots, z_n \in F$ with $z_0 = a$, $z_n = b$, and $d(z_{k-1}, z_k) < \varepsilon$ for $1 \leq k \leq n$. Notice the hypothesis of "open" is not given (or needed) here. HINT: Consider $F' = \{b \in F \mid \text{ there are points } z_0, z_1, \ldots, z_n \in F, \text{ for some } n \in \mathbb{N}, \text{ with } z_0 = a, z_n = b, \text{ and } d(z_{k-1}, z_k) < \varepsilon\}$. Show that F' is both open and closed in (F, d).

II.2.5. (b) If F is a set which satisfies the above property then F is not necessarily connected, even if F is closed. Give an example to illustrate this. Explain your answer. HINT: Think asymptotes.