Complex Analysis 1, MATH 5510, Fall 2017

Homework 6, Section III.1

Due: Friday, October 27 at 1:40

Show all work!!! Justify every claim and show all computations.

III.1.3. Prove that $\limsup(a_n + b_n) \leq \limsup a_n + \limsup b_n$ and $\liminf(a_n + b_n) \geq \liminf a_n + \liminf b_n$ for bounded sequences of real numbers $\{a_n\}$ and $\{b_n\}$. HINT: Recall that $L = \limsup a_n$ means that for all $\varepsilon > 0$, infinitely many a_n satisfy $a_n \in (L - \varepsilon, L + \varepsilon)$ and only finitely many a_n are greater than $L + \varepsilon$.

III.1.6d. Find the radius of convergence of the power series $\sum_{n=0}^{\infty} z^{n!}$. HINT: Use Theorem III.1.3 and ignore the coefficients which are 0.

III.1.7a. Show that the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n} z^{n(n+1)}$ is 1. HINT: The nth coefficient of this series is not $(-1)^n/n$. Give details.

III.1.7b. (Bonus) Discuss the convergence of the series in part (a) for $z = 1$, -1, and i. HINT: Consider the value of $\frac{(-1)^n}{n} i^{n(n+1)}$ for $n \pmod 4$. Consider the partial sums of the series s_n for n even and n odd. Show that the sequences of partial sums $\{s_2, s_2, s_6, \ldots\}$ and $\{s_1, s_3, s_5, \ldots\}$ both converge and have the same limit.