Complex Analysis 1, MATH 5510, Fall 2017

Homework 8, Section III.2, Solutions

Due: Friday, November 10 at 1:40

Show all work!!! Justify every claim and show all computations.

III.2.9. Suppose that $z_n, z \in G = \mathbb{C} \setminus \{z \mid z \leq 0\}$, $z_n = r_n e^{i\theta_n}$, and $z = re^{i\theta}$ where $\theta, \theta_n \in (-\pi, \pi)$. Prove that if $z_n \to z$ then $\theta_n \to \theta$ and $r_n \to r$. HINT: You need to argue geometrically. Let $\varepsilon > 0$. In the complex plane, $|r - r_n| < \varepsilon$ implies that z_n lies in the annulus $r - \varepsilon < |z| < r + \varepsilon$. The condition $|\theta - \theta_n| < \varepsilon$ means that z_n lies in the sector with sides $\theta - \varepsilon$ and $\theta + \varepsilon$.

III.2.14. Suppose $f : G \to \mathbb{C}$ is analytic and that G is open and connected. Prove that if $f(z)$ is real for all $z \in G$, then f is constant. HINT: Use the Cauchy-Riemann equations.

III.2.18. **(a)** Let $f : G \to \mathbb{C}$ and $g : G \to \mathbb{C}$ be branches of z^a and z^b, respectively, both based on the same branch of the logarithm on G. Prove that fg is a branch of z^{a+b}.

(b) Let $f : G \to \mathbb{C}$ and $g : G \to \mathbb{C}$ be branches of z^a and z^b, respectively, both based on the same branch of the logarithm on G. Prove that f/g is a branch of z^{a-b}.