Complex Analysis 1, MATH 5510, Fall 2017

Homework 9, Section III.3

Due: Tuesday, November 21 at 1:40

Show all work!!! Justify every claim and show all computations.

III.3.8. If $T(z) = \dfrac{az+b}{cz+d}$ is a Möbius transformation, then prove that $T(\mathbb{R}_\infty) = \mathbb{R}_\infty$ if and only if we can choose a, b, c, d to be real numbers. HINT: To show that a, b, c, d can be real, consider $T(0)$, $T(\infty)$, $T^{-1}(0)$, and $T^{-1}(\infty)$. To show that $T(\mathbb{R}_\infty) = \mathbb{R}_\infty$, use Theorem 3.14.

III.3.10. Let $D = \{z \mid |z| < 1\}$. Find all Möbius transformations T such that $T(D) = D$. These are the transformations which determine hyperbolic geometry in the Poincare disk model for hyperbolic geometry. HINT: Let $\alpha \in D$ be such that $T(\alpha) = 0$. Apply the Symmetry Principle and consider $T(\alpha^*)$.

III.3.26(a). Let $GL_2(\mathbb{C})$ be the multiplicative group of all invertible 2×2 matrices with entries in $\mathbb{C}$. Let $\mathcal{M}$ be the group of Möbius transformations. Define $\varphi : GL_2(\mathbb{C}) \to \mathcal{M}$ by

$$\varphi\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \frac{az+b}{cz+d}.$$

Prove that φ is a group homomorphism of $GL_2(\mathbb{C})$ onto $\mathcal{M}$. Find the kernel of φ.