APPLIED MATH II TEST 2
Spring 1997

NAME ______________________ STUDENT NUMBER ______________________

1. Answer three of the following:

a. If $f(x)$ has a Fourier series representation valid on the interval $x \in (-l, l)$, then what is the periodic extension, f_{per}, of f? On what interval is the Fourier representation of f equal to f_{per}?

b. Give a *geometric* argument to explain why if $\{X_1, X_2, \ldots,\}$ is a set of orthogonal functions on $[a, b]$ and $f(x) = \sum_{n=1}^{\infty} A_n X_n(x)$ on $[a, b]$, then $A_n = \dfrac{\int_a^b f(x)\overline{X_n(x)}\,dx}{\int_a^b |X_n(x)|^2\,dx}$ where the inner product is $(f, g) = \int_a^b f\overline{g}$.

c. Draw a picture of some piecewise continuous function of $[0, 1]$ (which is not continuous on $[0, 1]$) and seperately draw its Fourier Series.

d. State Parseval's Equality.

e. State the Maximum Principle for harmonic functions.

2. Do one of the following two:

a. Find a Fourier cosine series for $f(x) = |\sin(x)|$ on $(-\pi, \pi)$, if possible. If not possible, then explain. You may need the equation $\sin A \cos B = \frac{1}{2}\sin(A - B) + \frac{1}{2}\sin(A + B)$.

b. Find the Fourier sine series for $f(x) = x$ on $(0, l)$.

3. Do one of the following two:

a. The boundary conditions for an ODE on $[a, b]$ with eigenfunctions X_1 and X_2 are *symmetric* if $X_2'(x)X_1(x) - X_2(x)X_1'(x)\big|_a^b = 0$. Show that if X_1 and X_2 are eigenfunctions corresponding to distinct eigenvalues, then X_1 and X_2 are orthogonal.

b. The wave equation with homogeneous Neumann BCs on $[0, l]$ has general solution:

$$u(x,t) = \frac{1}{2}A_0 + \frac{1}{2}B_0 t + \sum_{n-1}^{\infty}\left(A_n \cos\frac{n\pi ct}{l} + B_n \sin\frac{n\pi ct}{l}\right)\cos\frac{n\pi x}{l}.$$

Use this information to solve:

$$\begin{aligned}
u_{tt} &= c^2 u_{xx} \text{ for } 0 < x < \pi, t > 0 \\
u_x(0,t) &= u_x(\pi, t) = 0 \\
u(x,0) &= 0 \\
u_t(x,0) &= \cos^2 x
\end{aligned}$$

HINT: $\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos(2x)$.

4. Do one of the following two:

a. Use orthogonality properties to show that if $\Phi(x) = \sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi x}{l}\right)$ on $(0, l)$, then

$$A_m = \frac{2}{l}\int_0^l \Phi(x) \sin\left(\frac{m\pi x}{l}\right)\, dx.$$

You might need the identity $\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos(2x)$.

b. Show that if the partial sums of the series $\sum_{n=1}^{\infty} f_n(x)$ converge uniformly on $[a, b]$ to the function f, then the partial sums converge in the L^2 sense as well. WARNING: simple pointwise convergence *does not* insure convergence of integrals!!!

5. HAND IN number 6.1.6.