Chapter 1. Linear Regression with One Predictor Variable

Note. In Part I, Simple Linear Regression, we consider the regression analysis of a response variable based on a single predictor variable. In Chapter 1, we introduce the basic ideas of regression analysis and consider the estimation of the parameters of linear regression models.

Section 1.1. Relations between Variables

Note. In this section we introduce the ideas of a "functional relation" and a "statistical relation."

Note. Throughout these notes, we consider only real numbers. So all variables under consideration are elements of $\mathbb{R}$. The domains and ranges of all functions are some subsets of $\mathbb{R}$.

Definition. A *functional relation* between two variables, X (the *independent variable*) and Y (the *dependent variable*) is a relation $Y = f(X)$ where f is some function mapping the possible values of X to the possible values of Y.

Note. From your mathematical background, you are deeply familiar with "functional relations." The definition of a "statistical relation," which we state next, is less rigorous.

Definition/Note. A *statistical relation* between two variables involves an *independent variable* (also called an *explanatory* or *predictor variable*) X, and a *dependent* or *response variable* Y. The relation between X and Y is "not a perfect one." The predictor variable is assumed to be measured precisely, but the response variable involves an uncertainty in its value. The statistical relation commonly involves a simple model (a linear model here) which approximates the scatter plot of data pairs (X, Y).

Note. An example of a statistical relation is given in Figure 1.2 where a scatter plot of several data points is given on the left and a (linear) statistical relation is given on the right. Notice that the statistical relation is "close to" several of the points, but does not pass exactly through any of the points.

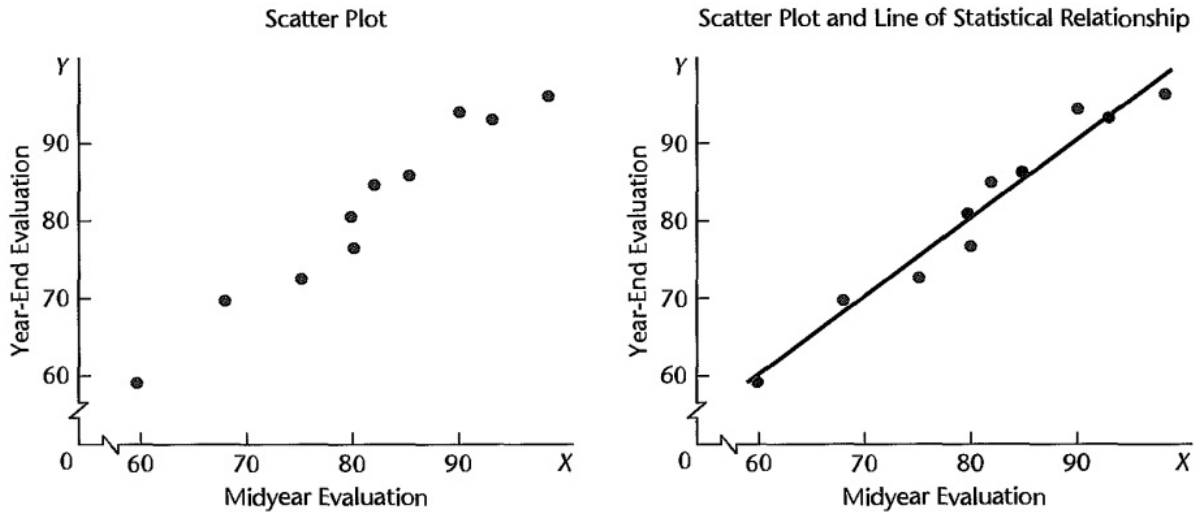

FIGURE 1.2 Statistical Relation between Midyear Performance Evaluation and Year-End Evaluation.