Chapter 5. Matrix Approach to Simple Linear Regression Analysis

Note. The first seven sections of this chapter covers ideas covered in a standard sophomore linear algebra class. See my online notes for Linear Algebra (MATH 2010) for details. There are also some online videos based on these notes. I also have online notes for graduate level Theory of Matrices (MATH 5090) and associated online videos. So we skip the material of sections 5.1 through 5.6, and give a list of results from 5.7 that will be referenced later in this chapter.

Note. Kutner et al. use bold faced fonts to indicate vectors and matrices. They use a prime to indicate the transpose of a matrix, so the transpose of matrix $\mathbf{A}$ is denoted $\mathbf{A}'$.

Note. The following properties of matrices are covered (with proofs given as exercises) in Linear Algebra (MATH 2010). See my online notes for this class on Section 1.3. Matrices and Their Algebra; notice in particular Theorem 1.3.A. For any matrices $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ of dimensions such that the following sums and products are defined, and for scalars k, k_1, k_2 we have:

(5.25) Commutivity Law of Addition: $\mathbf{A}+\mathbf{B}=\mathbf{B}+\mathbf{A}$. See Theorem 1.3.A(1).

(5.26) Associative Law of Addition: $(\mathbf{A}+\mathbf{B})+\mathbf{C}=\mathbf{A}+(\mathbf{B}+\mathbf{C})$. See Theorem 1.3.A(2).

(5.27) Associativity of Matrix Multiplication: $(\mathbf{AB})\mathbf{C} = \mathbf{A}(\mathbf{BC})$. See Theorem 1.3.A(8).

(5.28) Distributive Laws of Matrix Multiplication: $(\mathbf{A}+\mathbf{B})\mathbf{C} = \mathbf{AC}+\mathbf{BC}$ and $\mathbf{C}(\mathbf{A}+\mathbf{B}) = \mathbf{CA}+\mathbf{CB}$. See Theorem 1.3.A(10).

(5.29) Left and Right Distribution Laws: $k(\mathbf{A}+\mathbf{B}) = k\mathbf{A}+k\mathbf{B}$ and $(k_1+k_2)\mathbf{A} = k_1\mathbf{A}+k_2\mathbf{A}$. See Theorem 1.3.A(4,5).

Note. The following are also covered in Linear Algebra (MATH 2010), again in Section 1.3. Matrices and Their Algebra; notice in particular Note 1.3.B.

(5.30) $(\mathbf{A}')' = \mathbf{A}$. See Note 1.3.B (the first part).

(5.31) $(\mathbf{A}+\mathbf{B})' = \mathbf{A}'+\mathbf{B}'$. See Note 1.3.B (the second part).

(5.32) $(\mathbf{AB})' = \mathbf{B}'\mathbf{B}'$. See Note 1.3.B (the first third) and Exercise 1.3.32.

(5.33) $(\mathbf{ABC})' = \mathbf{C}'\mathbf{B}'\mathbf{A}'$. This follows by applying (5.32) twice.

Note. The following are covered in Linear Algebra (MATH 2010), in Section 1.5. Inverses of Matrices, and Linear Systems. We quote results from this source below.

(5.34) Inverses of Products: $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$. See Theorem 1.10.

(5.35) Inverses of Products: $(\mathbf{ABC})^{-1} = \mathbf{C}^{-1}\mathbf{B}^{-1}\mathbf{A}^{-1}$. This follows by applying (5.34) twice.

(5.36) $(\mathbf{A}^{-1})^{-1} = \mathbf{A}$. This follows from the uniqueness of an inverse matrix (see Theorem 1.9).

(5.37) $(\mathbf{A}')^{-1} = (\mathbf{A}^{-1})'$. This is exercise 1.5.24.

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