# Chapter 1. Linear Regression with One Predictor Variable

## Study Guide

The following is a brief list of topics covered in Chapter 1 of Michael Kutner, Christopher Nachtsheim, John Neter, and William Li's *Applied Linear Statistical Models*, 5th Edition (McGraw Hill, 2005). This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the proofs given in class and the homework problems.

**Section 1.1. Relations between Variables.**
Functional relation, independent variable, dependent variable, statistical relation, explanatory/predictor variable, dependent/response variable, scatterplot.

**Section 1.2. Regression Models and Theory Uses.**
The history of regression analysis (Tobias Mayer and the morion of the moon, Leonhard Euler and the orbits of Jupiter and Saturn, Roger Boscovich and the predecessor to the method of least squares, Laplace, Legendre and the orbits of comets, Gauss and the method of least squares, Francis Galton and statistical applications to biological data), the postulates of regression of data in two variables, regression function, regression curve, the pictorial representation of a regression model (Figure 1.4), causal role of the predictor variables, the desirability of having a theoretical framework for choice of the functional form, Lagrange polynomials, scope of a model, purposes of regression analysis (description, control, prediction), "correlation does not mean causation."

**Section 1.3. Simple Linear Regression Model with Distribution of Error Terms.**
Simple linear regression model $Y_1 = \beta_0 + \beta_1 X_i + \varepsilon_i$, simple, linear in parameters, linear in the predictor variable, first-order model, $E\{Y_i\} = \beta_0 + \beta_1 X_i$ (Note 1.3.A), $\sigma^2\{Y_i\} = \sigma^2$ (Note 1.3.B), the pictorial representation of a simple linear regression model (Figure 1.6), regression coefficients ($\beta_0$ and $\beta_1$), meaning of the parameters of simple linear regression (Figure 1.7), alternative fom of the simple linear regression model (Note 1.3.C).

**Section 1.4. Data for Regression Analysis.**

Observational data, experiments that vary the predictor variable, random manipulations of the explanatory variable, lutking variables, treatment, experimental units, completely randomized design.

**Section 1.5. Overview of Steps in Regression.**

Typical strategy for regression analysis (Figure 1.8), exploratory data analysis.

**Section 1.6. Estimation of Regression Function.**

Minimizing sum of squares (quantity $Q$, Note 1.6.A), illustration of minimizing $Q$ (Figure 1.9), the values $b_0$ and $b_1$ that minimize $Q$ (Theorem 1.6.A), The Gauss-Markov Theorem (Theorem 1.11), regression equation/regression line, example of the by-hand computation of $b_0$ and $b_1$ (Example 1.6.A), level of $X$, response $Y$, mean response $E\{Y\}$, fitted value $\hat{Y}_i$, observed value $Y_i$, example of the computation of $\hat{Y}$ (Example 1.6.B), alternative form of the estimated linear regression model (Note 1.6.B), residuals, example of the computation of residuals (Example 1.6.C), properties of residuals and the estimated regression function (Theorem 1.6.B).

**Section 1.7. Estimation of Error Terms Variance $\sigma^2$.**

Sample variance, deviation of observation $Y_i$, error sum of squares/residual sum of squares/SSE, error mean square/residual mean square/MSE, degrees of freedom (Note 1.7.A), estimator of $\sigma$ as $s = \sqrt{MSE}$, an example of the computation of $SSE$ and $MSE$ (Example 1.7.A).

**Section 1.8. Normal Error Regression Model.**

Normal error regression model (1.24), justification of the normal error assumption (Note 1.7.A), visualizing the normal distribution of $Y_i$ (Figure 1.15(d)), density of an observation, likelihood function, maximum likelihood estimators, agreement between least squares and maximum likelihood for the normal error regression model (Theorem 1.8.A).

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