Chapter 2. A Short Excursion into Matrix Algebra

Study Guide

The following is a brief list of topics covered in Chapter 2 of Härdle and Simar's *Applied Multivariate Statistical Analysis*, 5th Edition (Springer, 2019). This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the proofs given in class and the homework problems.

Section 2.1. Elementary Operations.

Matrix, vector, transpose, sum, difference, product of a scalar and a matrix, product of matrices, Properties of Matrix Operations (Theorem 2.1.A), linearly independent, linearly dependent, rank, trace, determinant, sign of a permutation, Properties of Determinants (Theorem 2.1.B), Properties of Transpose and Inverse (Theorem 2.1.C), minor matrix, cofactor, adjoint, generalized inverse, eigenvalue, eigenvector, characteristic polynomial, characteristic equation, computation of determinant and trace using eigenvalues (Theorem 2.1.E), properties of trace/rank/determinant (Theorem 2.1.F).

Section 2.2. Spectral Decompositions.

Jordan Decomposition (Theorem 2.1), orthogonal matrix, orthogonally diagonalizable, Fundamental Theorem of Real Symmetric Matrices, raising a matrix to a power (Note 2.2.A), Singular Value Decomposition (Theorem 2.2), singular values, spectral decomposition.

Section 2.3. Quadratic Forms.

Quadratic form, positive definite/positive semidefinite/indefinite quadratic form, expressing a quadratic form in terms of eigenvalues (Theorem 2.3), positive definiteness in terms of eigenvalues (Theorem 2.4), extrema of quadratic forms (Theorem 2.5).

Section 2.4. Derivatives.

Gradient and Hessian of $f : \mathbb{R}^p \to \mathbb{R}$, computation of gradient and Hessian (Exercise 2.5).

Section 2.5. Partitioned Matrices.

A matrix partitioned into four matrices, Algebraic Properties of Partitioned Matrices (Theorem 2.5.A), inverting a partitioned matrix (Theorem 2.5.B), determinants of partitioned matrices (Theorem 2.5.C), eigenvalues and eigenvectors of $\mathcal{AB}$ and $\mathcal{BA}$ (Theorem 2.6), rank of a product (Corollary 2.2).

Section 2.6. Geometrical Aspects.

Distance function, Euclidean distance, metric, isodistance curve E_d, Fundamental Theorem of Real Symmetric Matrices, principal aces/half-lengths of axes/rectangle surrounding E_d (Theorem 2.7), norm of a vector, unit vector, angle between vectors, projection, rotations, column space and null space, projection matrix, idempotent, relationships between projection/orthogonal complement/column space (Theorem 2.8).

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