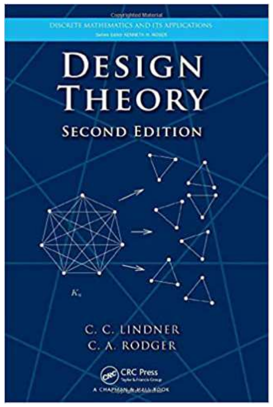
Design Theory

Chapter 6. Mutually Orthogonal Latin Squares

6.1. Introduction—Proofs of Theorems



Lemma 6.1.A

Lemma 6.1.A. If $\{L_1, L_2, \ldots, L_t\}$ is a collection of mutually orthogonal latin squares of order $n \geq 2$, MOLS(n), then the number of MOLS(n) satisfies $t \leq n - 1$.

Proof. By Note 6.1.B, we can put each of $L_1, L_2, \ldots, L_t$ is standard form with entry i in cell $(1, i)$ for $1 \leq i \leq n$ is each. The symbol 1 occurs in cell $(1, 1)$ of each of the latin squares, so (in the case $n \geq 2$) the cell $(2, 1)$ in each of them must be occupies by a symbol from $\{2, 3, \ldots, n\}$. If we consider any two of the latin squares superimposed, then the first rows give the ordered pairs $(1, 1), (2, 2), (3, 3), \ldots, (n, n)$. So cell $(2, 1)$ cannot be occupied by the same symbol x in different of the t latin squares, since the ordered pair (x, x) would occur when they are superimposed; but this pair is already present in the superimposed first rows. The number of possible MOLS(n) is then bounded by the number of possible entries in the $(2, 1)$ cells. Since these entries must be from set $\{2, 3, \ldots, n\}$ and none of the symbols can appear any two different of the MOLS(n), then the maximum number of MOLS(n) is $n - 1$. That is, $t \leq n - 1$. □