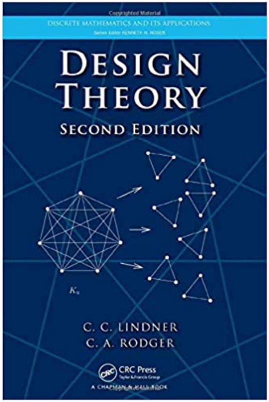
Design Theory

Chapter 6. Mutually Orthogonal Latin Squares

6.1. Introduction—Proofs of Theorems



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Lemma 6.1.A

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Proof. By Note 6.1.B, we can put each of $L_1, L_2, \ldots, L_t$ is standard form with entry i in cell $(1, i)$ for $1 \leq i \leq n$ is each. The symbol 1 occurs in cell $(1, 1)$ of each of the latin squares, so (in the case $n \geq 2$) the cell $(2, 1)$ in each of them must be occupies by a symbol from $\{2, 3, \ldots, n\}$. If we consider any two of the latin squares superimposed, then the first rows give the ordered pairs $(1, 1), (2, 2), (3, 3), \ldots, (n, n)$.

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