

2.6. λ -Fold Triple Systems in General

Note. In this section we use the results of Sections 1.3, 2.3, and 2.4 to give necessary and sufficient conditions for the existence of a λ -fold triple system. We use the triple systems for $\lambda = 1$ (in Section 1.3), $\lambda = 2$ (in Section 2.3), $\lambda \in \{3, 6\}$ (in Section 2.4), and repeated copies of triples.

Note. We will show the following conditions are necessary and sufficient for the existence of a λ -fold triple system.

Table 2.6.A

λ	spectrum of λ -fold triple systems
0 (mod 6)	all $v \neq 2$
1 or 5 (mod 6)	all $v \equiv 1$ or $3 \pmod{6}$
2 or 4 (mod 6)	all $v \equiv 0$ or $1 \pmod{3}$
3 (mod 6)	all odd v

Note. We specifically have the following. The proof is to be given in Exercise 2.6.7.

Theorem 2.6.1. A λ -fold triple system of order v exists if and only if v and λ satisfy the conditions given in Table 2.6.A.