# Chapter 1. Steiner Triple Systems
## Study Guide

The following is a brief list of topics covered in Chapter 1 of Lindner and Rodger's *Design Theory* Second Edition, Discrete Mathematics and Its Applications Series, CRC Press (2008). This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the proofs given in class and the homework problems.

**Section 1.1. The Existence Problem.**
Steiner triple systems, triples, order of a Steiner triple system, examples, interpretation of a Steiner triple system as a decomposition of $K_n$ into 3-cycles, a $STS(7)$ (Example 1.1.A), history of Steiner triple systems (W. S. B. Wool-House, Jakob Steiner, and Thomas Kirkman), necessary conditions for the existence of a Steiner triple system (Lemma 1.1.A).

**Section 1.2. $v \equiv 3 \pmod 6$: The Bose Construction.**
Raj Chandra Bose, eugenics/Francis Galton/Karl Pearson/Ronald Fisher, latin square, quasigroup, idempotent latin square, commutative latin square, the construction of idempotent commutative quasigroups of odd order based on $\mathbb{Z}_{2n+1}$, the Bose construction (Type 1 and Type 2 triples), a Steiner triple system exists of all orders $v \equiv 3 \pmod 6$ (Theorem 1.2.A), example of the Bose construction.

**Section 1.3. $v \equiv 1 \pmod 6$: The Skolem Construction.**
Thoralf Skolem, half idempotent latin square/quasigroup, the Skolem construction (Type 1, Type 2, and Type 3 triples), a Steiner triple system exists of all orders $v \equiv 1 \pmod 6$ (Theorem 1.3.A), necessary and sufficient conditions for existence of a Steiner triple system (Theroem 1.3.B), example of the Skolem construction.

**Section 1.4. $v \equiv 5 \pmod 6$: The $6n + 5$ Construction.**
Pairwise balanced design (PBD), blocks, order of a PBD, the $6n + 5$ construction of a PBD with one block of size 5 and all other blocks of order 3, example of the $6n + 5$ construction.

**Section 1.5. Quasigroups with Holes and Steiner Triple Systems.** Quasigroup with holes $H$, renaming the blocks of size three in a PBD($v$) (Note 1.5.A, Example 1.5.2, and Exercise 1.5.A), the construction of a commutative quasigroup from a STS and possibly an idempotent commutative quasigroup of order 5 (Theorem 1.5.5 and Example 1.5.6), The Quasigroup with Holes Construction of a STS and PBD (Figure 1.7, Example 1.5.11, and Example 1.5.13).

**Section 1.6. The Wilson Construction.** 1-factor of a graph, 2-factor of a graph, 1-factorization, $r$-factor, deficiency graph of $(\mathbb{Z}_n, +)$ (and Note 1.6.A), wheel and spokes (in the sense used in Lindner and Rodger), 1-factorization of a wheel (Example 1.6.1), biwheel, biwheels can be 1-factored into three 1-fators (Lemma 1.6.A), The Deficiency Graph Algorithm (and Example 1.6.3), Wilson's Construction of Steiner triple systems (and Example 1.6.4).

**Section 1.7. Cyclic Steiner Triple Systems.**
Isomorphism (in general), permutations as a product of disjoint cycles, cyclic permutation, automorphism of a Steiner triple system, cyclic Steiner triple system, the history of Heffter's difference problems and Rose Peltesohn, difference triple, Heffter's First and Second Difference Problems, base block, short orbit blocks, use of Heffter's difference problem to give necessary and sufficient conditions for a cyclic Steiner triple system (Theorem 1.7.6), example of a cyclic Steiner triple system, Peltesohn's solution (Appendix A).

**Section 1.8. The $2n + 1$ and $2n + 7$ Constructions.**

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