

Chapter 2. λ -Fold Triple Systems

Study Guide

The following is a brief list of topics covered in Chapter 2 of Lindner and Rodger's *Design Theory* Second Edition, Discrete Mathematics and Its Applications Series, CRC Press (2008). This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the proofs given in class and the homework problems.

Section 2.1. Triple Systems of Index $\lambda > 1$.

λ -fold triple system, triples, order, 2-fold triple systems as decompositions of $2K_n$, multiset.

Section 2.2. The Existence of Idempotent Latin Squares.

Transversal of a latin square, stripping the transversal T of a quasigroup with examples and Lemma 2.2.A, the existence of idempotent quasigroups (Theorem 2.2.3).

Section 2.3. 2-Fold Triple Systems.

2-fold triple system and examples, order, "spectrum," the two constructions of 2-fold triple systems (Notes 2.3.A and 2.3.B), necessary and sufficient conditions for the existence of a 2-fold triple system (Theorem 2.3.7).

Section 2.4. Mendelsohn Triple Systems.

Digraphs and arcs (or "directed edges"), complete directed graph D_n , directed triple, Mendelsohn triple system MTS , interpretation of MTS as an arc-disjoint decomposition of D_n , the two constructions of a Mendelsohn triple system (Notes 2.4.A and 2.4.B), necessary and sufficient conditions for the existence of a Mendelsohn triple system (Theorem 2.4.7).

Supplement. Directed and Hybrid Triple Systems.

Directed triples/transitive triples, directed triple system DTS , interpretation of DTS as an arc-disjoint decomposition of D_n , the two constructions of a Mendelsohn triple system (Notes A and B), necessary and sufficient conditions for the existence of a directed triple system (Theorem DTS-HTS A), necessary and sufficient conditions for the existence of a λ -fold directed triple system (Theorem DTS-HTS B), c -hybrid triple system $c-HTS$, oriented triple system/ordered triple system, interpretation of $c-HTS$ as an arc-disjoint decomposition of D_n , necessary and sufficient conditions

for the existence of a c -HTS(v) triple system (Theorem DTS-HTS C), necessary and sufficient conditions for the existence of an oriented triple system (Theorem DTS-HTS D).

Supplement. Mixed Triple Systems.

Steiner triple system, Mendelsohn triple system, directed triple system, mixed graph, complete mixed graph, T_i -mixed triples, mixed triple system, hybrid mixed triple systems (open problem), Hartman and Mendelsohn's "Last of the Triple Systems," Jum's "Last of the Mixed Triple Systems" (partially open problem), packing and covering with mixed triples (partially open problem), λ -fold mixed triple systems (open problem), Beeler and Meadows' partial orientations of P_4 and S_3 decompositions of complete mixed graphs, Culver and Gardner's partial orientations of S_6 decompositions of complete mixed graphs, mixed hexagon systems (open problem).

Section 2.5. $\lambda = 3$ and 6.

The construction of a 3-fold triple system (Note 2.5.A), the construction of a 6-fold triple system (Note 2.5.B), necessary and sufficient conditions for the existence of 3-fold and 6-fold triple systems (Theorem 2.5.7).

Section 2.6. λ -Fold Triple Systems in General.

Necessary and sufficient conditions for the existence of λ -fold triple system (Theorem 2.6.1).

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