DIFFERENTIAL EQUATIONS II, TEST I
11:15-12:30

NAME _____ STUDENT NUMBER _____

Each problem is worth 14 points. Show all work. Be neat and use equal signs where applicable. Remember, you are not only trying to find the answer, but you are also trying to convince me that you know what you are doing! No calculators.

1. If $\mathbf{A} = \begin{pmatrix} 6 & -3 \\ 2 & 1 \end{pmatrix}$ then what are the eigenvalues and eigenvectors of $\mathbf{A}$?

2. If $\mathbf{A} = \begin{pmatrix} 6 & -3 \\ 2 & 1 \end{pmatrix}$ then find a diagonal matrix $\mathbf{D}$ and a matrices $\mathbf{T}$ and $\mathbf{T}^{-1}$ such that $\mathbf{A} = \mathbf{TDT}^{-1}$.

3. Prove that if $\lambda = 0$ is an eigenvalue of $\mathbf{A}$ then $\mathbf{A}$ is singular.

4. Solve $\mathbf{x}' = \begin{pmatrix} 6 & -3 \\ 2 & 1 \end{pmatrix} \mathbf{x}$.

5. Solve $\mathbf{x}' = \begin{pmatrix} 4 & -1 \\ 1 & 2 \end{pmatrix} \mathbf{x}$.

6. Find a fundamental matrix $\mathbf{\Phi}(t)$ satisfying $\mathbf{\Phi}(0) = \mathbf{I}$ for the system

$$\mathbf{x}' = \begin{pmatrix} 6 & -3 \\ 2 & 1 \end{pmatrix} \mathbf{x}.$$

7. Given that $\begin{pmatrix} \sin x \\ x \end{pmatrix}$ is a particular solution to

$$\mathbf{x}' = \begin{pmatrix} 6 & -3 \\ 2 & 1 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 3x + \cos x - 6\sin x \\ x + 1 - 2\sin x \end{pmatrix},$$

what is the general solution to this system?