Theorem 1.6.1

**Theorem 1.6.1.** If $\varphi(t)$ is a solution of $\ddot{x} = A\dot{x}$ where $A$ is a constant matrix (with real entries) then $\text{Re}(\varphi(t))$ and $\text{Im}(\varphi(t))$ are also solutions.

**Proof.** Let $\varphi(t) = \bar{u}(t) + i\bar{v}(t)$. Then

$$
\varphi'(t) = \bar{u}'(t) + i\bar{v}'(t) = A\varphi(t) = A\bar{u}(t) + iA\bar{v}(t).
$$

So $\bar{u}'(t) = A\bar{u}(t)$ and $\bar{v}'(t) = A\bar{v}(t)$, so that $\bar{u}(t)$ and $\bar{v}(t)$ are also solutions to $\dot{x} = A\bar{x}$, as claimed.