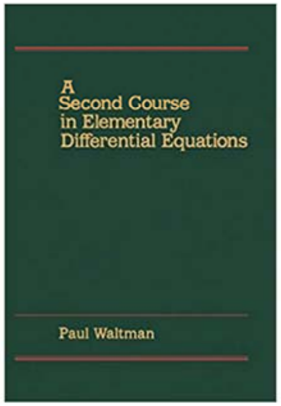
Advanced Differential Equations

Chapter 1. Systems of Linear Differential Equations

Section 1.6. The Constant Coefficient Case: Complex and Distinct Eigenvalues—Proofs of Theorems



Theorem 1.6.1

Theorem 1.6.1. If $\vec{\varphi}(t)$ is a solution of $\vec{x}\,' = A\vec{x}$ where A is a constant matrix (with real entries) then $\mathrm{Re}(\vec{\varphi}(t))$ and $\mathrm{Im}(\vec{\varphi}(t))$ are also solutions.

Proof. Let $\vec{\varphi}(t) = \vec{u}(t) + i\vec{v}(t)$. Then

$$\vec{\varphi}\,'(t) = \vec{u}\,'(t) + i\vec{v}\,'(t) = A\vec{\varphi}(t) = A\vec{u}(t) + iA\vec{v}(t).$$

So $\vec{u}\,'(t) = A\vec{u}(t)$ and $\vec{v}\,'(t) = A\vec{v}(t)$, so that $\vec{u}(t)$ and $\vec{v}(t)$ are also solutions to $\vec{x}\,' = A\vec{x}$, as claimed. □