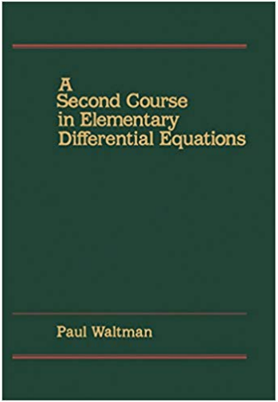
Advanced Differential Equations

Chapter 1. Systems of Linear Differential Equations

Section 1.6. The Constant Coefficient Case: Complex and Distinct Eigenvalues—Proofs of Theorems



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Proof. Let $\vec{\varphi}(t) = \vec{u}(t) + i\vec{v}(t)$. Then

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So $\vec{u}\,'(t) = A\vec{u}(t)$ and $\vec{v}\,'(t) = A\vec{v}(t)$, so that $\vec{u}(t)$ and $\vec{v}(t)$ are also solutions to $\vec{x}\,' = A\vec{x}$, as claimed. □

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