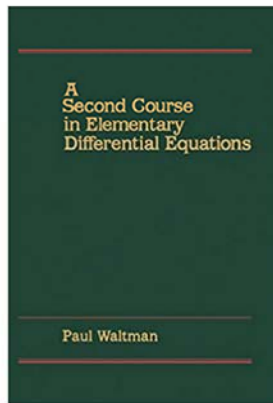


Advanced Differential Equations

Chapter 1. Systems of Linear Differential Equations Section 1.8. General Linear Systems—Proofs of Theorems



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Theorem 1.8.1

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Theorem 1.8.1. If $\vec{x}(t)$ is a solution of

$$\vec{y}' - A\vec{y} = \vec{e}(t) \quad (8.2)$$

($\vec{e}(t)$ is called a *forcing term*), then any solution $\vec{\Psi}(t)$ of (8.2) can be written as

$$\vec{\Psi}(t) = \Phi(t)\vec{c} + \vec{x}(t)$$

where Φ is a fundamental matrix for

$$\vec{y}' - A\vec{y} = \vec{0} \quad (8.3)$$

and \vec{c} is a constant.

Proof. Let $L[\vec{y}] = \vec{y}' - A\vec{y}$ and let $L[\vec{\Psi}(t)] = \vec{e}(t)$. Then $L[\vec{\Psi}(t) - \vec{x}(t)] = \vec{e}(t) - \vec{e}(t) = \vec{0}$. So $\vec{\Psi}(t) - \vec{x}(t)$ is a solution of (8.3). By Theorem 1.3.4, $\vec{\Psi}(t) - \vec{x}(t) = \Psi(t)\vec{c}$ and so $\vec{\Psi}(t) = \Psi(t)\vec{c} + \vec{x}(t)$. \square

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Theorem 1.8.2

Theorem 1.8.2

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$$\vec{x}(t) - \Phi(t) \int_{t_0}^t \Phi^{-1}(\tau) \vec{e}(\tau) d\tau$$

is a solution of $\vec{y}' = \vec{e}(t)$.

Proof. Well,

$$\vec{x}'(t) = \Phi'(t) \int_{t_0}^t \Phi^{-1}(\tau) \vec{e}(\tau) d\tau + \Phi(t) \Phi^{-1}(t) \vec{e}(t).$$

Now $\Phi'(t) = A(t)\Phi(t)$ so

$$\vec{x}' = A(t)\Phi(t) \int_{t_0}^t \Phi^{-1}(\tau) \vec{e}(\tau) d\tau + \vec{e}(t) = A(t)\vec{x}(t).$$

\square

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