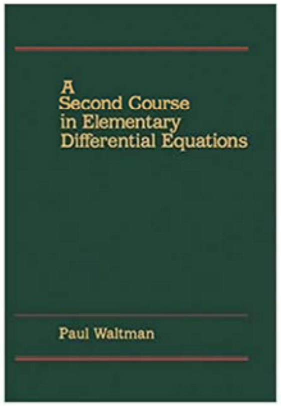
Advanced Differential Equations

Chapter 2. Two-Dimensional Autonomous Systems

Section 2.4. Critical Points of General 2-D Linear Systems—Proofs of Theorems



Example. Exercise 2.2.4

Example. (Exercise 2.4.4.) If $D = T^{-1}AT$ is a diagonal matrix, then the columns of T are eigenvectors of T are eigenvectors of A and the nonzero entries of D are eigenvalues of A.

Proof. Suppose $D = T^{-1}AT$ where D is diagonal. Then $AT = TD$. Suppose $T = [t_{ij}]$ and $D = [d_{ij}]$. Then the (i,j) entry of TD is $\sum_{k=1}^n t_{ik}d_{kj} = t_{ij}d_{jj}$. So the jth column o fTD is $d_{jj}\begin{bmatrix} t_{1j} \\ t_{2j} \\ \vdots \\ t_{nj} \end{bmatrix}$. That is, the jth column of TD is d_{jj} times the jth column of T. So the jth column of AT is d_{jj} times the jth column of T. Therefore, the jth column of T is an eigenvector of A with eigenvalue d_{jj}. □