Example. (Exercise 2.2.4.) If \( D = T^{-1}AT \) is a diagonal matrix, then the columns of \( T \) are eigenvectors of \( T \) are eigenvectors of \( A \) and the nonzero entries of \( D \) are eigenvalues of \( A \).

**Proof.** Suppose \( D = T^{-1}AT \) where \( D \) is diagonal. Then \( AT = TD \).

Suppose \( T = [t_{ij}] \) and \( D = [d_{ij}] \). Then the \((i,j)\) entry of \( TD \) is

\[
\sum_{k=1}^{n} t_{ik} d_{kj} = t_{ij} d_{jj}.
\]

So the \( j \)th column of \( TD \) is \([d_{ij} \ldots d_{ij}]^T\). That is, the \( j \)th column of \( TD \) is \( d_{jj} \) times the \( j \)th column of \( T \). So the \( j \)th column of \( AT \) is \( d_{jj} \) times the \( j \)th column of \( T \). Therefore, the \( j \)th column of \( T \) is an eigenvector of \( A \) with eigenvalue \( d_{jj} \). \( \square \)