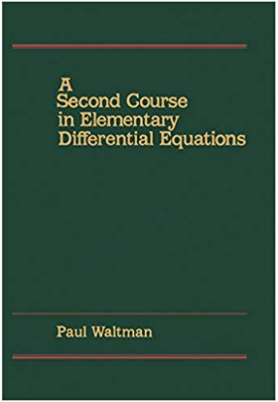
Advanced Differential Equations

Chapter 2. Two-Dimensional Autonomous Systems

Section 2.4. Critical Points of General 2-D Linear Systems—Proofs of Theorems



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Proof. Suppose $D = T^{-1}AT$ where D is diagonal. Then $AT = TD$. Suppose $T = [t_{ij}]$ and $D = [d_{ij}]$. Then the (i, j) entry of TD is $\sum_{k=1}^{n} t_{ik}d_{kj} = t_{ij}d_{jj}$. So the jth column o fTD is $d_{jj}\begin{bmatrix} t_{1j} \\ t_{2j} \\ \vdots \\ t_{nj} \end{bmatrix}$.

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