APPLIED MATH 1 TEST 1 - Fall 1996

NAME ______________________________ STUDENT NUMBER ________________________

1. Answer each of the following.

   (a) (b) State the *definition* of an "ordinary differential equation." Include the definition of "order" and "solution."

   (c) Give two conditions equivalent to "matrix A is invertible."

   (d) State the definition of a linear operator and give an example of one (other than the one given in problem 3 below).

   (e) Suppose λ is an eigenvalue for matrix A with corresponding eigenvector $\mathbf{c}$. Give a solution to the system $\mathbf{y}' = A\mathbf{y}$.

2. Do one of the following two:

   (a) *Prove* the product of two diagonal matrices is diagonal.

   (b) Prove that if λ is an eigenvalue of matrix A (with real entries) with corresponding eigenvector $\mathbf{c}$, then $\overline{\lambda}$ is an eigenvector of A with corresponding eigenvector $\overline{\mathbf{c}}$.

3. Do one of the following two:

   (a) If A and B are $n \times n$ matrices such that $AB = BA$, prove $e^A e^B = e^{A+B}$ (be clear on where you are using the fact that A and B commute).

   (b) Prove that $L[\mathbf{y}] = \mathbf{y}' - A\mathbf{y}$ is a linear operator.

4. Find a fundamental matrix for:

$$\mathbf{y}' = \begin{pmatrix} 2 & -3 \\ 3 & 2 \end{pmatrix} \mathbf{y}.$$

5. Find a fundamental matrix for:

$$\mathbf{y}' = \begin{pmatrix} 3 & -1 \\ 1 & 1 \end{pmatrix} \mathbf{y}.$$