Section 1.7. The Constant Coefficient Case: The Putzer Algorithm

Note. In this section we give a method for generating linearly independent solutions for systems where we have repeated eigenvalues.

Theorem. (**Cayley-Hamilton Theorem**, from Linear Algebra.)
Let A be an $n \times n$ matrix and let p be the polynomial $p(\lambda) = \det(A - \lambda\mathcal{I}) = (\lambda - \lambda_1)(\lambda - \lambda_2)\cdots(\lambda - \lambda_n)$. Then $p(A) = 0$.

Theorem 1.7.1. Putzer Algorithm.
Let A be an $n \times n$ matrix with eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$. Then

$$e^{At} = \sum_{j=0}^{n-1} R_{j+1}(t) P_j \qquad (7.1)$$

where $p_0 = \mathcal{I}$,

$$P_j = \prod_{k=1}^{j} (A - \lambda_k \mathcal{I}) \text{ for } j = 1, 2, \ldots, n-1 \qquad (7.2)$$

and $R_1(t), R_2(t), \ldots, R_n(t)$ is the solution of

$$\begin{cases} R_1' = \lambda_1 R_1 \\ R_j' = R_{j-1} + \lambda_j R_j,\ j = 2, 3, \ldots, n \\ R_1(0) = 1 \\ F_j(0) = 0,\ j = 2, 3, \ldots, n. \end{cases} \qquad (7.3)$$