Chapter 2. Two-Dimensional Autonomous Systems

Section 2.1. Introduction

Note. In this section we state one theorem. It is an "existence and uniqueness" result. Notice that it also implies continuity with respect to initial conditions.

Theorem 2.1.1. Let $f(x,y), g(x,y)$ be continuously differentiable (i.e., all first order partials are continuous). Then there is a unique solution of the IVP

$$\begin{cases} x' = f(x,y) \\ y' = g(x,y) \end{cases} \qquad (1.2)$$

$$x(t_0) = \alpha, y(t_0) = \beta$$

where $' = d/dt$, valid on interval $(t_0 - \gamma, t_0 + \gamma) = I$. (Notice that a solutions is a pair of functions $x(t)$ and $y(t)$.) If the solution is denoted by $x(t,\alpha,\beta), y(t,\alpha,\beta)$ then for a fixed $t \in I$, $x(t,\alpha,\beta)$ and $y(t,\alpha,\beta)$ are continuous functions of α and β.

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