

Shepley L. Ross *Introduction to Ordinary Differential Equations*

Chapter 1. Differential Equations and Their Solutions

1.2. Solutions

1.2.1. Show that each of the functions defined in Column I is a solution of the corresponding DE in Column II on every interval $a < x < b$ of the x -axis.

I	II
(a) $f(x) = x + 3e^{-x}$	$\frac{dy}{dx} + y = x + 1$
(b) $f(x) = 2e^{3x} - 5e^{4x}$	$\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 12y = 0$
(c) $f(x) = e^x + 2x^2 + 6x + 7$	$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 4x^2$
(d) $f(x) = \frac{1}{1+x^2} = (1+x^2)^{-1}$	$(1+x^2)\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = 0$

1.2.2(a). Show that $x^3 + 3xy^2 = 1$ is an implicit solution of the DE $2xy\frac{dy}{dx} + x^2 + y^2 = 0$ on the interval $x \in (0, 1)$.

1.2.3. (a) Show that every function f defined by $f(x) = (x^3 + c)e^{-3x}$, where c is an arbitrary constant, is a solution of the differential equation $\frac{dy}{dx} + 3y = 3x^2e^{-3x}$.

(b) Show that every function f defined by $f(x) = 2 + ce^{-2x^2}$, where c is an arbitrary constant, is a solution of the differential equation $\frac{dy}{dx} + 4xy = 8x$.

1.2.5. (a) For certain values of the constant m the function f defined by $f(x) = e^{mx}$ is a solution of the differential equation $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 12y = 0$. Determine all such valued of m .
HINT: Use the Rational Roots Theorem (see Theorem B of Appendix 2).

(b) For certain values of the constant n the function g defined by $g(x) = x^n$ is a solution of the differential equation $x^3\frac{d^3y}{dx^3} + 2x^2\frac{d^2y}{dx^2} - 10x\frac{dy}{dx} - 8y = 0$. Determine all such valued of n .
HINT: Use the Rational Roots Theorem (see Theorem B of Appendix 2).