Shepley L. Ross *Introduction to Ordinary Differential Equations*

Chapter 1. Differential Equations and Their Solutions

1.3. Initial-Value Problems, Boundary-Value Problems and Existence of Solutions

1.3.1. Show that $y = 4e^{2x} + 2e^{-3x}$ is a solution of the initial-value problem

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0, \; y(0) = 6, \; y'(0) = 2.$$

Is $y = 2e^{2x} + 4e^{-3x}$ also a solution of this problem? Explain why or why not.

1.3.3. Given that every solution of $\dfrac{d^2y}{dx^2} - \dfrac{dy}{dx} - 12y = 0$ may be written in the form $y = c_1e^{4x} + c_2e^{-3x}$, for some choice of the arbitrary constants c_1 and c_2, solve the given initial value problems.

(a) $\dfrac{d^2y}{dx^2} - \dfrac{dy}{dx} - 12y = 0$, $y(0) = 5$, $y'(0) = 6$.

1.3.5. Given that every solution of $x^3\dfrac{d^3y}{dx^3} - 3x^2\dfrac{d^2y}{dx^2} + 6x\dfrac{dy}{dx} - 6y = 0$ may be written in the form $y = c_1x + c_2x^2 + c_3x^3$, for some choice of the arbitrary constants c_1, c_2, and c_3, solve the given initial value problem consisting of the above differential equation plus the three conditions $y(2) = 0$, $y'(2) = 2$, $y''(2) = 6$.